SIMULATIONS FOR SPECTRAL DISTRIBUTIONS OF SOME LARGE BLOCK-MATRICES

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1. Definitions

(1) Let

$$L_k =: \begin{bmatrix} A_n & B_n & C_n & B_n & C_n & \dots & C_n & B_n \\ B_n & A_n & B_n & C_n & B_n & \dots & B_n & C_n \\ C_n & B_n & A_n & B_n & C_n & \dots & C_n & B_n \\ B_n & C_n & B_n & A_n & B_n & \dots & B_n & C_n \\ C_n & B_n & C_n & B_n & A_n & \dots & C_n & B_n \\ \vdots & & & & & \ddots & & \\ C_n & B_n & C_n & B_n & A_n & \dots & A_n & B_n \\ B_n & C_n & B_n & C_n & B_n & \dots & B_n & A_n \end{bmatrix}$$

be a $k \times k$ block-matrix. A_n, B_n and C_n are $n \times n$ Wigner matrices which are independent but not necessarily identically distributed. Simulations for the spectral distribution of 100 L_k with $n = 100$ for

$k = 2, 3, 4, 5, 6, 7, 8, 9$ are given in Figure 1 and 2. The heights are up to normalizing constants.

(2) Let

$$T_k = \begin{bmatrix} A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & A_n^{(4)} & A_n^{(5)} & \dots & A_n^{(k-1)} & A_n^{(k)} \\ A_n^{(2)} & A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & A_n^{(4)} & \dots & A_n^{(k-2)} & A_n^{(k-1)} \\ A_n^{(3)} & A_n^{(2)} & A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & \dots & A_n^{(k-3)} & A_n^{(k-2)} \\ A_n^{(4)} & A_n^{(3)} & A_n^{(2)} & A_n^{(1)} & A_n^{(2)} & \dots & A_n^{(k-4)} & A_n^{(k-3)} \\ A_n^{(5)} & A_n^{(4)} & A_n^{(3)} & A_n^{(2)} & A_n^{(1)} & \dots & A_n^{(k-5)} & A_n^{(k-4)} \\ \vdots & & & & & \ddots & & \\ A_n^{(k-1)} & A_n^{(k-2)} & A_n^{(k-3)} & A_n^{(k-4)} & A_n^{(k-5)} & \dots & A_n^{(1)} & A_n^{(2)} \\ A_n^{(k)} & A_n^{(k-1)} & A_n^{(k-2)} & A_n^{(k-3)} & A_n^{(k-4)} & \dots & A_n^{(2)} & A_n^{(1)} \end{bmatrix}$$

be a $k \times k$ Toeplitz block-matrix. $A_n^{(1)}, A_n^{(2)}, \dots, A_n^{(k)}$ are $n \times n$ Wigner matrices which are independent but not necessarily identically distributed. Simulations for the spectral distribution of 100 T_k with $n = 100$ for $k = 3, 4, 5, 6, 7, 11$ are given in Figure 3. The heights are up to normalizing constants.

(3) Let

$$W_2 = \begin{bmatrix} A_n & B_n \\ B_n & A_n \end{bmatrix}$$

be a 2×2 Toeplitz block-matrix where A_n is $n \times n$ Wigner matrix and B_n is $n \times n$ Wishart matrix. Simulation for the spectral distribution

of 100 W_2 with $n = 100$ is given in Figure 4. The heights are up to normalizing constants.

(4) Let

$$Y_k = \begin{bmatrix} A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & A_n^{(4)} & A_n^{(5)} & \dots & A_n^{(k-1)} & A_n^{(k)} \\ A_n^{(2)T} & A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & A_n^{(4)} & \dots & A_n^{(k-2)} & A_n^{(k-1)} \\ A_n^{(3)T} & A_n^{(2)T} & A_n^{(1)} & A_n^{(2)} & A_n^{(3)} & \dots & A_n^{(k-3)} & A_n^{(k-2)} \\ A_n^{(4)T} & A_n^{(3)T} & A_n^{(2)T} & A_n^{(1)} & A_n^{(2)} & \dots & A_n^{(k-4)} & A_n^{(k-3)} \\ A_n^{(5)T} & A_n^{(4)T} & A_n^{(3)T} & A_n^{(2)T} & A_n^{(1)} & \dots & A_n^{(k-5)} & A_n^{(k-4)} \\ \vdots & & & & & \ddots & & \\ A_n^{(k-1)T} & A_n^{(k-2)T} & A_n^{(k-3)T} & A_n^{(k-4)T} & A_n^{(k-5)T} & \dots & A_n^{(1)} & A_n^{(2)} \\ A_n^{(k)T} & A_n^{(k-1)T} & A_n^{(k-2)T} & A_n^{(k-3)T} & A_n^{(k-4)T} & \dots & A_n^{(2)T} & A_n^{(1)} \end{bmatrix}$$

be a $k \times k$ Toeplitz block-matrix. $A_n^{(1)}$ is an $n \times n$ Wigner matrix and $A_n^{(2)}, \dots, A_n^{(k)}$ are $n \times n$ non-symmetric random matrices which are independent but not necessarily identically distributed. Here, A^T is the matrix transpose of A . Simulations for the spectral distribution of 100 Y_k with $n = 100$ for $k = 2, 3, 4, 5$ are given in Figure 5. The heights are up to normalizing constants.

2. Figures

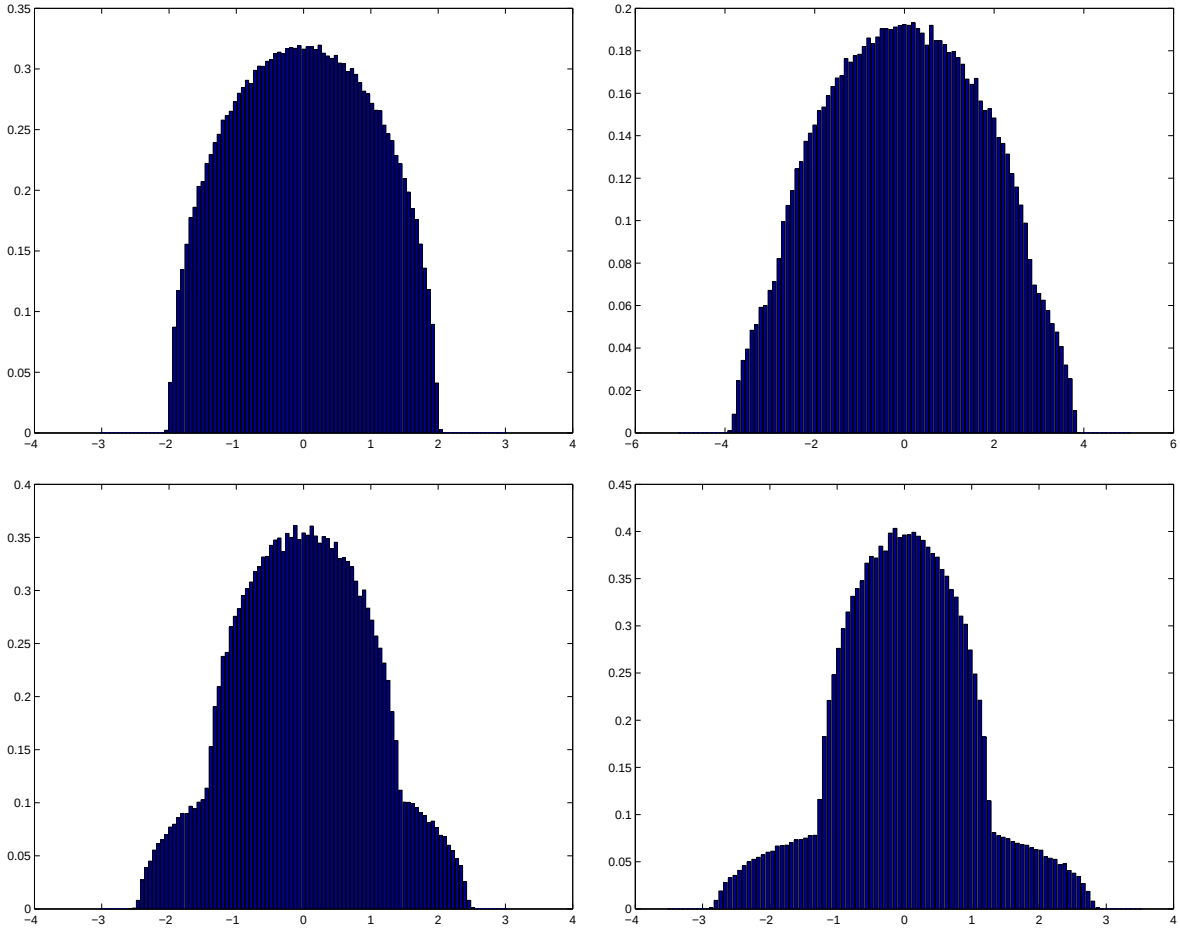

FIGURE 1. Simulation of the limiting spectral distributions of (from right to left): L_2, L_3, L_4 and L_5

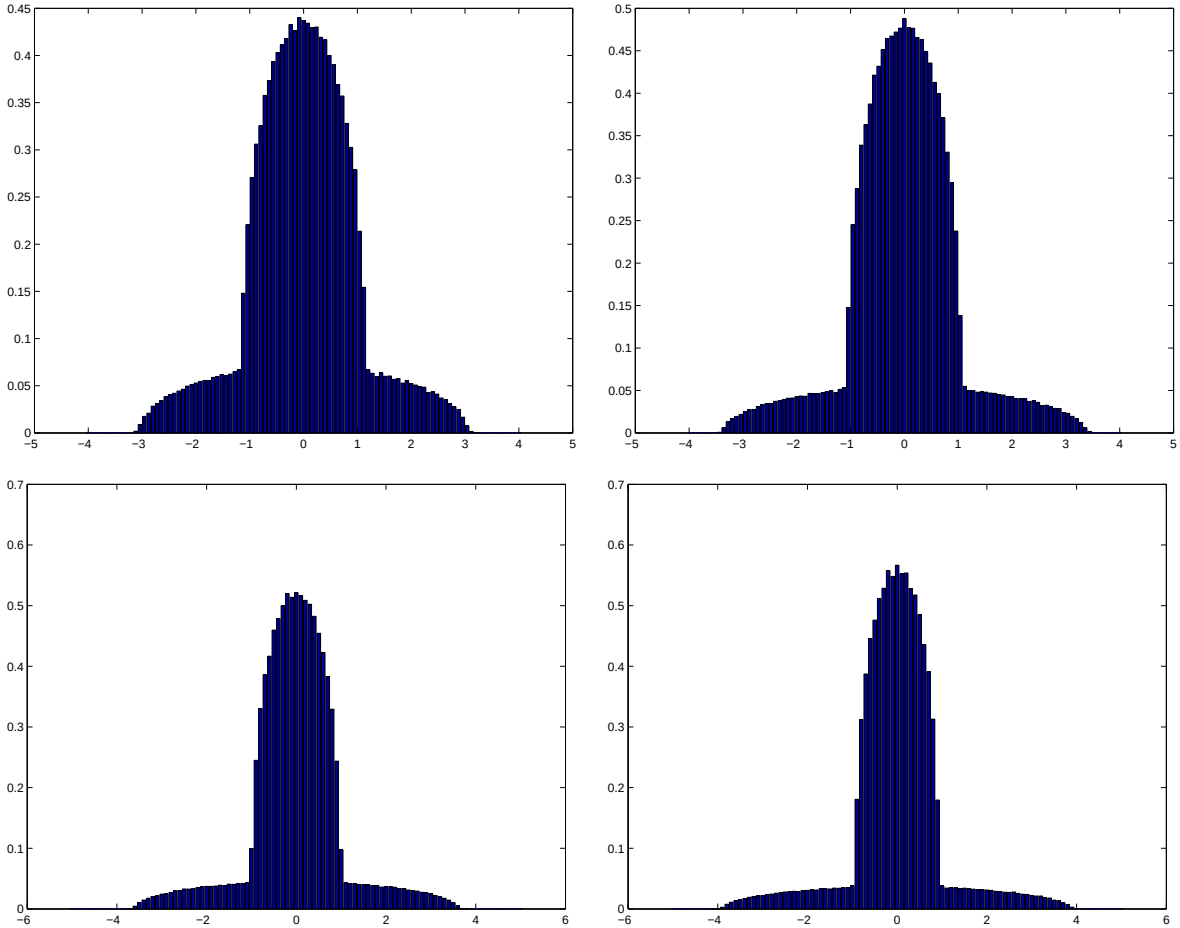

FIGURE 2. Simulation of the limiting spectral distributions of (from right to left): L_6, L_7, L_8 and L_9

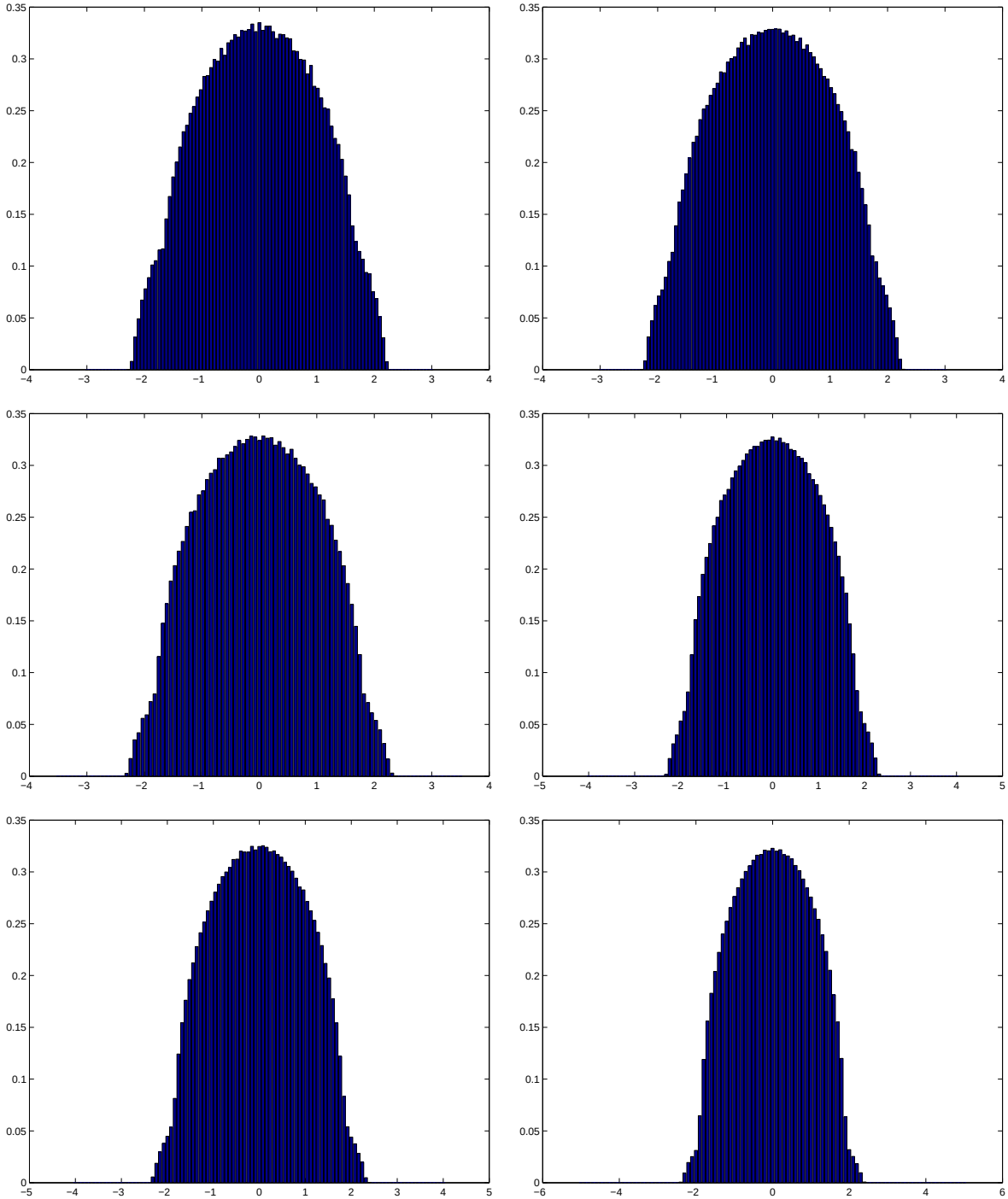

FIGURE 3. Simulation of the limiting spectral distributions of (from right to left): T_3, T_4, T_5, T_6, T_7 and T_{11}

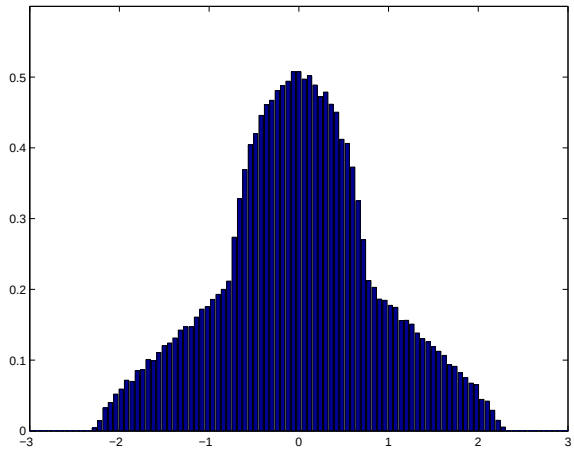

FIGURE 4. Simulation of the limiting spectral distribution of W_2

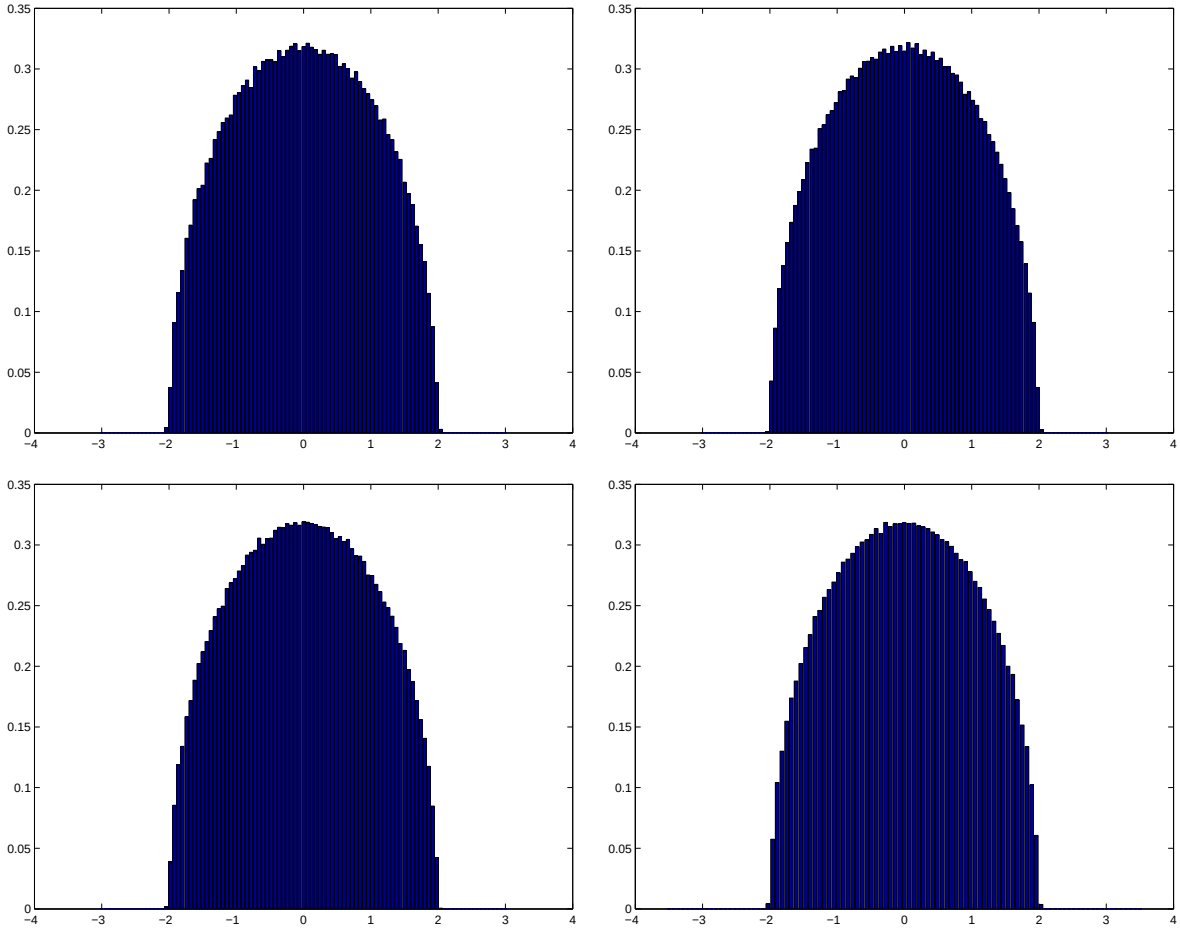

FIGURE 5. Simulation of the limiting spectral distributions of (from right to left): Y_2, Y_3, Y_4 and Y_5