

### Problem 2009-1

Consider the differential equation

$$x''(t) + a(t)x^{2n+1}(t) = 0, \quad 0 \leq t < \infty.$$

The given function  $a(t)$  is continuously differentiable, and  $a(t) \geq a_0 > 0$  for all  $t$ ;  $n$  is a positive integer.

(i) If  $a'(t)$  has only finitely many changes of sign, prove that any solution  $x(t)$  is bounded.

(ii) If one does not assume that  $a'(t)$  has only finitely many changes of sign, is  $x(t)$  necessarily bounded?