

# Old and New Results on the Student t-statistic

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The Student t-statistic was proposed by William Gossett in 1908 to construct confidence intervals and perform statistical tests about the mean of a normal-like population based on a finite sample, when the population standard deviation is unknown. At the time, Gossett was working for the Guinness Brewery and published his work under the pseudonym, “Student”, in order to keep his “frivolous” mathematical activities secret from his employer. Not long afterwards, R. A. Fisher put the Student t-statistic in its present form and first rigorously derived its distribution for any finite sample under the assumption that the population is normal. (Actually a normal population exists only in theory.) This led to the computation of the tables of critical values for Student t-statistic, which one finds in elementary statistics textbooks. The critical values converge quickly to those of the standard normal distribution as the sample size increases to infinity. Therefore Student t-statistic critical values are often not provided in textbooks for sample sizes much larger than 30. In fact, this convergence to the standard normal critical values will hold true even when the population is not normal, as long as the underlying population has a finite variance. Of course the rate will depend on the underlying distribution. Only recently was the question resolved: When is the Student t-statistic asymptotically standard normal? The solution to this problem has revealed some unexpected connections with other old problems. The speaker will discuss his participation in the answer to this question and its unexpectedly relation to a problem motivated by the arcsine law.