Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

1. KenKen is a popular new math puzzle and here is one for you to solve. The rules are as follows:
   - ☞ Each square in the puzzle below is filled in with one of the digits $1, 2, 3, 4$.
   - ☞ Each digit occurs exactly once in each row and exactly once in each column.
   - ☞ The numbers in each heavily outlined set of squares, called "cages,"must combine (in any order) to produce the number in the top corner, using the mathematical operation indicated.
   - ☞ A number can be repeated within a cage as long as it is not in the same row or column.
   - ☞ At the back of the test, another copy of the puzzle appears. Use that copy to do your work, use the copy below just for your answer, but submit everything.

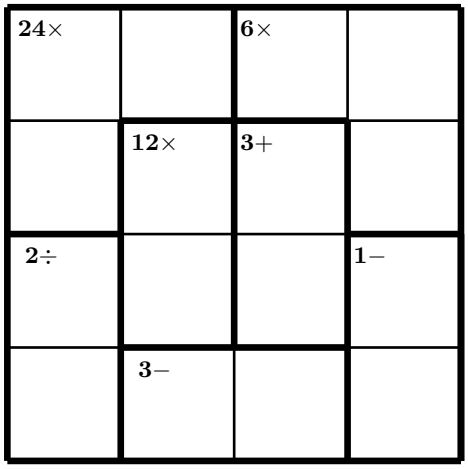

Figure 1: Do your work on the other copy, use the above just for your answer. Submit both pages.

Solution:
Solution in rows:

4	2	3	1
3	4	1	2
1	3	2	4
2	1	4	3

2. The tens digit of a two-digit number exceeds the units digit by 3. If the order of the digits is reversed, the new number is one less than half the original number. Find the two numbers.

Solution: The candidates are $41, 52, 63, 74, 85, 96$. The answer must be even since the first number is the double of one more than the second number. Only 52 fits the criteria. The two numbers are 52 and 25.

3. Charlie ate several pieces of candy on Halloween. Each day after Halloween he ate six fewer pieces than the day before. His candy lasted exactly five days, and he ate 100 pieces of candy in total. How many did he eat on the third day?

Solution:
Let c be the number of pieces of candy Charlie ate on the middle day. Then the first day he ate $c+12$ and the second day he ate $c+6$. The fourth day he ate $c-6$ and the last day he ate $c-12$. All together he ate $5c$ pieces of candy, so on the third day he ate 20.

4. A cube is painted yellow on all sides, and then it is sliced up to make 125 equal smaller cubes. How many of these smaller cubes have paint on at least two sides? How many have paint on at least one side?

Solution: If we take the outer layer of small cubes off the entire cube, what is left is a $3 \times 3 \times 3$ cube, which has 27 unpainted cubes. Therefore there are $125 - 27 = 98$ cubes with paint on at least one side. On each of the faces, there are $3 \times 3 = 9$ cubes that are on the interior of the face and have paint on only one side. So all together there are $6 \times 9 = 54$ cubes that are painted on exactly one side. Then there are $98 - 54 = 44$ cubes that are painted on at least two sides.

5. How many scalene triangles have perimeter of length ≤ 12, where the length of each side is an integer? (A triangle is *scalene* if no two sides have the same length.)

Solution: Let x, y, z be the lengths of the three sides, in increasing order. Then $xx < y < z, x+y+z \leq 12$, and $x+y > z$. This last condition means $z \leq 5$ and $x > 1$, since $x+y > z > y$. This limits the possibilities to consider, and shows that the only solutions are $(2,3,4), (2,4,5), (3,4,5)$.