Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

1. KenKen is a popular new puzzle type. Here is one for you to solve. The rules are as follows:

   - Each square in the puzzle below is filled in with one of the digits $1, 2, 3, 4, 5, 6$.
   - Each digit occurs exactly once in each row and exactly once in each column.
   - The numbers in each heavily outlined set of squares, called "cages," must combine (in any order) to produce the number in the top corner, using the mathematical operation indicated.
   - Cages with just one square should be filled in with the target number in the top corner.
   - A number can be repeated within a cage as long as it is not in the same row or column.
   - At the back of the test, another copy of the puzzle appears. Use that copy to do your work, use the copy below just for your answer, but submit everything.

2×		**1−**		**15+**	
	8+		**11+**		
13+				**15×**	
	600×	**11+**		**4×**	
		4	**36×**		
	3÷				

Figure 1: Do your work on the other copy, use the above just for your answer. Submit both pages.

Solution: Solution in rows:

2	1	3	4	5	6
1	3	2	5	6	4
4	6	1	2	3	5
3	4	5	6	2	1
6	5	4	3	1	2
5	2	6	1	4	3

2. There are 750 students at Sosmart High School. Of these, 300 students are taking Spanish, 200 are taking U.S. History, and 150 are taking Physics. 340 students are not taking any of these three courses. There are 110 students who are taking both Spanish and US History and 90 who are taking both Spanish and Physics. There are 40 students who are actually taking all three subjects. How many students are taking both Physics and US History?

Solution: There are $750 - 340 = 410$ students taking at least one of the subjects. If S is the set of students taking Spanish, H is the set of students taking US History, and P is the set of students taking Physics, then we are looking for the number of students who are in the intersection of P and H, that is, $|P \cap H|$. The "inclusion-exclusion rule" tells us that

$$|S \cup H \cup P| = |S| + |H| + |P| - |S \cap P| - |S \cap H| - |P \cap H| + |S \cap P \cap H|.$$

This is because $|S| + |H| + |P|$ counts the number of students taking all three subjects three times, and the number of students taking exactly two of the subjects two times each. So we have
$410 = 300 + 200 + 150 - 90 - 110 - |P \cap H| + 40$, and $|P \cap H| = 650 - 200 + 40 - 410 = 80$.

3. A square and an equilateral triangle have the same area. What is the ratio of the perimeter of the triangle to the perimeter of the square?

Solution:
Let A denote the area of the square and triangle. Let s be the length of one side of the square, and b be the base of the equilateral triangle. Then $s^2 = A = \frac{\sqrt{3}}{4}b^2$, so $\frac{\sqrt[4]{3}}{2}b = s$. The perimeter of the equilateral triangle is $3b$ and of the square is $4s = 2\sqrt[4]{3}b$, and the ratio is $\frac{3}{2\sqrt[4]{3}} = \frac{\sqrt[4]{27}}{2}$.

4. If $a + b = 2$ and $a^2 + b^2 = 3$, determine $a^3 + b^3$.

Solution:
We have $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. Now $(a+b)^2 = a^2 + 2ab + b^2$, so $ab = \frac{1}{2}((a+b)^2 - (a^2+b^2)) = \frac{1}{2}(2^2 - 3) = \frac{1}{2}$. Then

$$a^3 + b^3 = (a+b)[a^2 + b^2 - ab] = 2[3 - \frac{1}{2}] = 5.$$

5. Amy can mow her yard in an hour, but it takes her sister Bonnie an hour and a half to mow the same yard.
   (a) How long does it take them to mow the yard if they both work together? (Assume they each still mow at the same rate.)
   (b) If Bonnie mows for 15 minutes before Amy comes to help, how long will Amy and Bonnie have to mow together in order to finish the job?
   (c) In this case, what fraction of the yard will Bonnie have done?

Solution:

1. Together they can mow $1+2/3 = 5/3$ yard per hour, so it will take them $3/5$ hour, or 36 minutes, to mow the yard.
2. If Bonnie mows for 15 minutes before Amy comes to help, she will have mowed $2/3 \cdot 1/4 = 1/6$ of the yard. There will be $5/6$ of the yard to finish, and it will take them half an hour, or 30 minutes.
3. In this case, Bonnie will have done $1/6$ of the yard before Amy arrives, and in the next 30 minutes, she will mow $2/3 \cdot 1/2 = 1/3$ more of the yard, so she will have done half the yard. To check, notice that Amy mows for 30 minutes, so she has done half the yard also.