

## Advanced Questions #6-10

Instructions: Write your answers in the blue book. Remember that you must explain your solutions. Even correct answers without complete justifications may receive little credit. Also, even if you can't completely solve a problem, you should carefully explain what you have discovered about the problem since some partial credit may be awarded for your work. Have Fun!

6. Archimedes, Euclid, Fermat, and Gauss had a mathematics competition. Archimedes said "I did not finish 1st or 4th." Euclid said "I did not finish 4th." Fermat said "I finished 1st." Gauss said "I finished 4th."

There were no ties in the competition and exactly three of the mathematicians were telling the truth about where they finished. Who was 1st and who was 4th? Justify your answers!

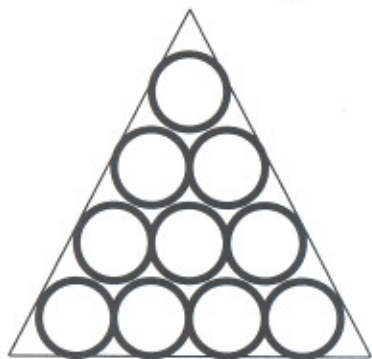
7. From the four numbers  $x_1 < x_2 < x_3 < x_4$  one can select a pair in 6 ways. Suppose that each pair has a different sum and that the 4 smallest sums are 1,2,3, and 4. What are the possible values of  $x_4$ ?
8. Define the function  $f$  for  $n = 3, 4, \dots$  by

$$f(n) = \log_2(3) \times \log_3(4) \times \cdots \times \log_{n-1}(n).$$

What is the value of  $\sum_{2 \leq k \leq 10} f(2^k)$ ?

9. If  $y = x^4 + x$  what is  $\frac{dx}{d(y^2)}$  when  $x = 1$ ?

10. An equilateral triangle is completely filled with  $n$  rows of congruent, tangent circles. The case  $n = 4$  is shown in the figure:



What is the value of the limit

$$\lim_{n \rightarrow \infty} \frac{\text{area of circles}}{\text{area of triangle}}$$