Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

1. (10 points) How many tons of rain fall over a square mile of land when one inch of rain falls? ☼☼////

   Hint: 1 mile = 5280 feet, 1 cubic foot of water weighs 62.5 lbs and 2000 lbs= 1 ton.

Solution: There are $C = (5280)^2 \times (1/12) = 2,323,200$ cu. ft. of rain. The weight in lbs. is $62.5 \times C$ and in tons is $62.5 \times C/2000 = 72,600$.

2. (10 points) The following pictures (figures 1 & 2) 'prove' $64 = 65$.
   (a) (3 points) Pretend that $65 = 64$ is true, now think of a reason why any two numbers are equal? Hint: Let n be any number, $65n = 64n$, so $n =$?
   (b) What is wrong with the figures?
   Hint: In figure 2, carefully label x & y axes so that the (dotted) diagonal has equation $x/13+y/5 = 1$.

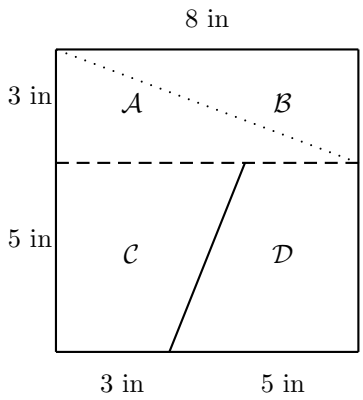


Figure 1: Area($\mathcal{A}$) + Area($\mathcal{B}$) + Area($\mathcal{C}$) + Area($\mathcal{D}$) = 64 sq. in. See question 2.

Solution:

(a) If $64 = 65$ then $0 = 1$, so multiply by n and $0 = n$. So there is only one number 0.

(b) The dotted line in figure 1 is supposed to be 3 units high or $x = 5, y = 3$, should satisfy the equation $x/13 + y/5 = 1$, but $5/13 + 3/5 = 64/65 < 1$.

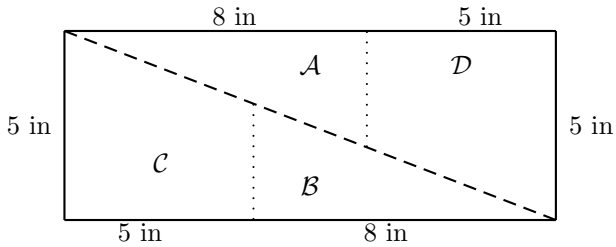
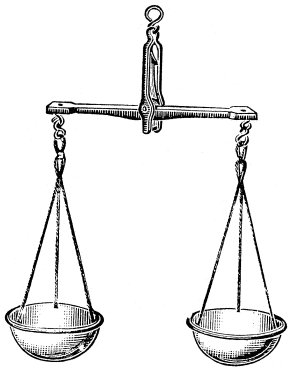
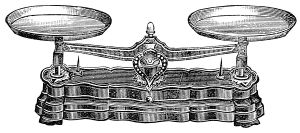
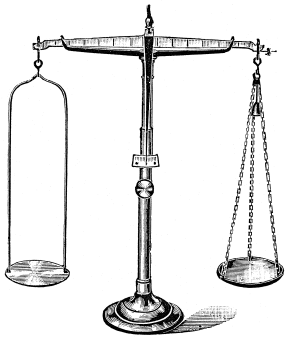


Figure 2: Area($\mathcal{A}$) + Area($\mathcal{B}$) + Area($\mathcal{C}$) + Area($\mathcal{D}$) = $5 \times 13 = 65$ sq. in. See question 2.







3. (10 points) There are 9 balls which all weigh the same except for one which is a little heavier. Using a balance (as illustrated above) at most 2 times, find a way to identify the heavy ball.

Solution: Pick any 3 balls, call them group A and put them on one side of the balance, pick another set of 3 balls, call them group B and put them on the other side of the balance. The unused balls we call group C. If the balance stays in balance, the heavy ball is in C, if the balance dips to one side, that is the side with the heavy ball. Now you know which group the heavy ball is among, use almost the same idea, but this time the groups have size 1, on the next weighing to find the heavy ball.

4. (10 points) If robot 1 can wash and polish the cafeteria floor in 3 hours and robot 2 can wash and polish the floor in 5 hours, how long will it take both robots, to wash and polish the cafeteria floor?

Solution: Assume the area of the cafeteria floor is F square feet. Robot 1 washes and polishes $F/3$ sq.ft/hr and robot 2 washes and polishes $F/2$ sq.ft/hr. Thus the robots wash and polish

$$\frac{F}{3} + \frac{F}{2}\text{sq.ft/hr.}$$

$x =$ the number of hours for both robots to wash and polish the cafeteria floor.

Thus

$$x \times \left(\frac{F}{3} + \frac{F}{2}\right)\frac{\text{sq.ft}}{\text{hr.}} = x \cdot \left(\frac{(3+2)\cdot F}{3\cdot 2}\right)\frac{\text{sq.ft}}{\text{hr.}} = F \text{ sq.ft} .$$

So

$$x = \frac{6\cdot \cancel{F}}{5\cdot \cancel{F}}\frac{\text{hr.}}{\cancel{\text{sq.ft}}}\cdot\frac{\cancel{\text{sq.ft.}}}{1} = \frac{6}{5}\text{ hr.} \tag{1}$$

5. The *simple interest formula* is a very important one. P is the amount saved, it is called the principal, the annual interest rate is r and it must be entered as a decimal [1] and t is number of years the money is invested. Then the balance after t years is

$$P(1+r)^t.$$

A sum of money at simple interest amounts to \$2556.02 in 2 years and to \$2767.10 in 4 years.

(a) (5 points) What is the interest rate and

(b) (5 points) what is the principal?

Solution: We know $P(1+r)^2 = 2556.02$ & $P(1+r)^4 = 2767.1$. Take the ratio and, we have

$$(1+r)^2 = \frac{P(1+r)^4}{P(1+r)^2} = \frac{2767.1}{2556.02} = 1.08258151345\cdots .$$

So

$$r = \sqrt{1.08258151345\cdots} - 1 = .040471774459\cdots .$$

Making the interest rate a percent , we have 4.047% annual interest. Also

$$P = \frac{2556.02}{(1+r)^2} = \frac{2556.02}{1.08258151345} = \$2361.09.$$

Now we can check our work, by using the formula for an interest rate of $r = .04047$, $P = 2361.09$ for 4 years,

$$2361.09(1.04047)^4 = 2767.13.$$

Notice this is 3 cents higher and your answers might be different also, these differences are probably due to round-off.

[1] For example, 6.0350% is

$$r = (1/100) \times (6.0350) = .06035$$