Beginning Level: **Part 1** The 22nd UC Math Bowl March 3, 2007

Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

1. (10 points) Henry took a 1700 mile trip. For the first 15 hours, he travelled at 55 MPH, but then slowed down to 45 MPH for the rest of the trip. What was his average speed for the whole trip?

Solution: How many hours did it take Henry to complete the trip? In the first 15 hours, he did $15 \times 55 = 825$ miles, leaving 875 miles. At 45 MPH, this will take 19.44 hours. So in total he will drive for 34.44 hours for an average speed of $1700/34.44 = 49.3548$ MPH.

2. (10 points) If nine thousand nine hundred and nine dollars is properly written \$9,909, how should twelve thousand twelve hundred and twelve dollars be written?[1]

Solution: \$12,000+\$1,200+\$12=\$13,212.

3. (10 points) In the sequence of numbers $1, 3, 2, \cdots$ each term after the first two is defined to be equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of this sequence.

Solution: $a_4 = 2 - 3 = -1, a_5 = -1 - 2 = -3, a_6 = -3 - (-1) = -2$. Note the sum of the first 6 terms are 0. Show they repeat, so the sum from 1 to 100 is the sum from 97 to 100 or 1+3+2+(-1)=5.

4. (10 points) The dots in the square grid (see figure 1) are equally spaced 1 cm apart both vertically and horizontally. How many straight line segments of length 5 can be made joining two of these dots?

Hint: Find 3-4-5 right triangles.

Solution: We look for right triangles with legs of length 3 and 4, then the hypotenuse has length 5. The line does not have to pass thru 5 vertices, all it has to do is connect 2 of the vertices and have length 5. Figure 1 illustrate such a triangle and figure 2 shows all of the hypothenuses. There are 8 of them.

[1] *The Colossal Book of Short Puzzles and Problems*, by M. Gardner(W.W.Norton & Co:2006), p. 61. A copy of this book will be given away as a door prize at 1 PM today.

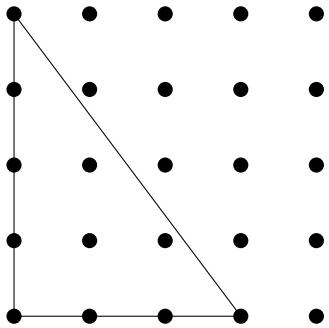

Figure 1: A $3-4-5$ right triangle.

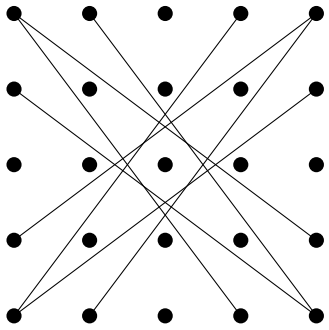

Figure 2: Just the hypotenuses.

5. (10 points) We will use algebra to explain some magic[2] tricks. The mathemagician asks Bob

   1. to think of a one or two digit number,
   2. double it,
   3. add 12,
   4. divide the total by 2,
   5. subtract the original number.

   She then seems to read Bob's mind by telling him that he is thinking of the number 6. Did she use magic or math? What is the algebra behind this trick?

   Hint: Let x be Bob's initial number.

Solution: $x \dashrightarrow 2x \dashrightarrow 2x + 12 \dashrightarrow \frac{2x+12}{2} = x + 6 \dashrightarrow x + 6 - x = 6$

[2] *Secrets of Mental Math*, by A. Benjamin, M. Shermer(New York:Three Rivers Press:2006), p.199. Two copies of the book will be given away as door prizes at 1 PM today.