Beginning Level: **Part 2** — The $21^{\text{th}}$ UC Math Bowl — March 4, 2006

Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

1. (10 points) Fred was born before 1900, and he died in 1950. When he died, his age was 1/29th the year of his birth. How many years old was he in 1910?

> **Solution:** Freds age at the time of his death was 1/29th the year of his birth. His age is also the difference between the year of his death and the year of his birth. So
> $$1950 - B = \frac{B}{29},$$
> where B is the year of his birth. Solving this equation gives $B = 1885$. Fred was born in 1885. In 1910, Fred was twenty-five years old.

2. (10 points) How many cubic inches of dirt are there in a hole that is one foot deep, 2 feet wide and six feet long?

> **Solution:** Since the dirt has been removed, there are 0 cubic inches of dirt in the hole. B ut 12 cubic feet of dirt was removed and so $12 \times 12^3 = 12^4 = 20,736$ cubic inches.

3. (10 points) Prove [1] there is no positive integer $n$ such that
$$1324^n + 731^n = 1961^n.$$
Hint: What are the possible values of the ones digit?

> **Solution:** : The ones digit of $1324^n$ is 4 or 6 & the ones digit of $731^n$ & $1961^n$ is 1. Notice the ones digit on the left side is 4 or 6 plus 1, which means the ones digit on the left is 5 or 7. But the ones digit on the right is 1.

[1] The problem is a special case of Fermat's Last Theorem which states for any integer $n > 2$ there are no integers $a, b, c$ with
$$a^n + b^n = c^n,$$
and $abc \neq 0$. This was demonstrated by Andrew Wiles about 10 years ago. For the whole wonderful story see the book by Simon Singh *Fermat's Last Theorem*.

4. (10 points) If 20 people go to a party and they all shake hands with one another, how many handshakes will there be?

**Solution:** Each person shakes hands with 19 people. That would be $20 \times 19$ handshakes, but this way we count every handshacke twice. Therefore, the total number of handshackes is $10 \times 19 = 190$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 7 | | 6 | | | | | |
| | | | | 1 | | 9 | | |
| | | | 5 | | | | | |
| | | | | | | | | |
| 6 | | 4 | | | | 2 | 9 | 8 |
| | 8 | | | | | | 4 | |
| | | | 9 | | 6 | | | |
| | | | | | | | | |
| | 3 | | 2 | | 5 | | | |

Figure 1: Pseudo-Sudoku

5. (10 points) (Pseudo-Sudoku) The diagram above does not qualify as a Sudoku puzzle, because it can be completed in several different ways. But there is enough information to determine 3 of the grid squares. Remember the rules of Sudoku, each of the nine $3\times3$ super squares, each row and each column must contain the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$.

   We will refer to the individual squares using two numbers. The first number tells the row (rows go across) and the second tells the column (columns go up and down). So there is a 6 in $(1, 4), (5, 1)$ & $(7, 6)$.

   (a) Explain why only a 1 or a 5 can be in $(5, 2)$.

   (b) If a 5 is put in square $(5, 2)$ what numbers must be put in squares $(5, 4), (5, 5)$ & $(5, 6)$? Can you determine exactly which number goes in each of these squares?

   (c) You are still putting a 5 in $(5, 2)$. Why can the fourth column not be filled in according to the rules of Sudoku?

   (d) You should now be able to identify 2 of the squares and the numbers that must be put in them. What is the row and column addresses of these squares and what values must be put in them?

   (e) There is a third square. What is its row and column address and what value must be put in it?

**Solution:**

(a) Look at the numbers in the fifth row, second column and left middle supercell. Only 1 & 5 are missing.

(b) If a 5 is put in $(5, 2)$ then 1,3 & 7 are needed to complete row. You can not determine which number goes in each cell.

(c) To complete the fourth column we need 1, 3, 4, 7, 8. Now one of the numbers 1, 3 or 7 will be in $(5, 4)$, but then the other 2 of these must be put in $(2, 4)$ and $(8, 4)$. So to finish column 4, 4 & 8 must be put in the 2 open cells $(4, 4)$ and $(6, 4)$. But neither 4 nor 8 can be put in $(6, 4)$, because a number can occur just once in each row, column and supercell.

(d) By (c) we know that a 1 must be put in $(5, 2)$ and 5 must be put in $(5, 5)$.

(e) 1 must be put in $(6, 4)$, because of reasons very similar to the answer for (c).