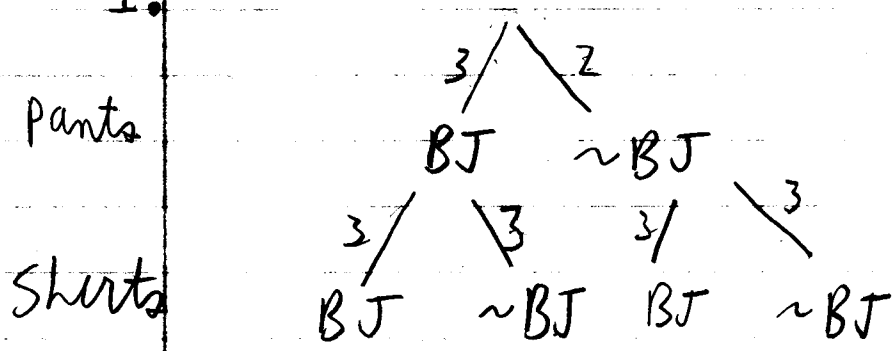
Level 3: Part 2 2005

1.



There are 30 possible outfits 9 of them are entirely BJ's. So the probability of entirely BJ's is

$$9/30 = \frac{3}{10} = .3$$

2. $10! = 3{,}628{,}800$. There are 2 zeros

one of them is from the 10 & the other from the 2 & the 5.

$100! = 100 \cdots 95 \cdots 90 \cdots 85 \cdots 80 \cdots 75 \cdots 70 \cdots 65 \cdots 60$

$\cdots 55 \cdots 50 \cdots 45 \cdots 40 \cdots 35 \cdots 30 \cdots 25 \cdots 20 \cdots 15 \cdots 10 \cdots 5 \cdots 1$

There are lots of 2 between 1 & 100 & there are 24 5's between 1 & 100. Note 25, 50 75, 100 are multiples of 5^2

L3P2

So there are 24 zeros in $100! \simeq 9.3 \cdot 10^{157}$

3. Let x = # of sheets of paper
y = # of envelopes.

Ann: She wrote y letters and she used y sheets of paper, but she had 50 sheets left over. So

$$y + 50 = x$$

Sue: She wrote $\frac{x}{3}$ letters & she used $(x/3)$ envelopes. So

$$y = \frac{x}{3} + 50$$

So we have
$$x - y = 50$$
$$-\frac{x}{3} + y = 50$$

So $\frac{2x}{3} = 100$ $\quad x = \frac{3}{2} \cdot 100 = 150$ & $y = 50$.

4. $1 + 2 + \cdots + 100 = (100+1) + (99+2) + \cdots + (51+50)$

$= 50(101) = 5050$

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

5.

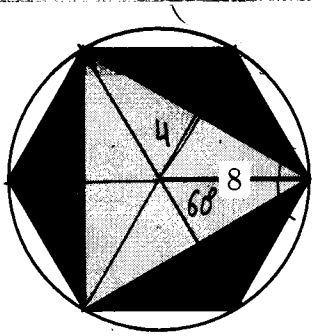
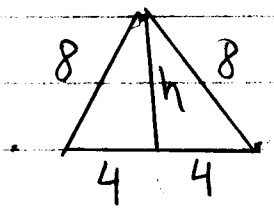


Figure for Problem 5

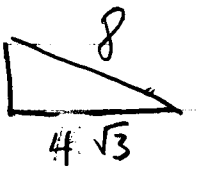
Divide the hexagon into 6 triangles as shown Note the central ∡ is 60°. Since each triangle is isosceles the base angles are equal. So they are also 60°. Thus each of the 6 triangles is equilateral. Now let be one of the 6 triangles



So $h^2 + 4^2 = 8^2$ $h^2 = 48 = 16 \cdot 3$

$h = 4\sqrt{3}$

The area of the hexagon is $6\left(\frac{1}{2} \cdot 8 \cdot 4\sqrt{3}\right)$ $= 96\sqrt{3}$. The equilateral triangle is made up of 6 small triangles



So the equilateral triangle has area

$$6 \cdot \frac{1}{2} \cdot 4 \cdot 4\sqrt{3} = 48\sqrt{3}$$

So the dark gray region has area

$$= 96\sqrt{3} - 48\sqrt{3} = 48\sqrt{3}$$

L3 P4 Good