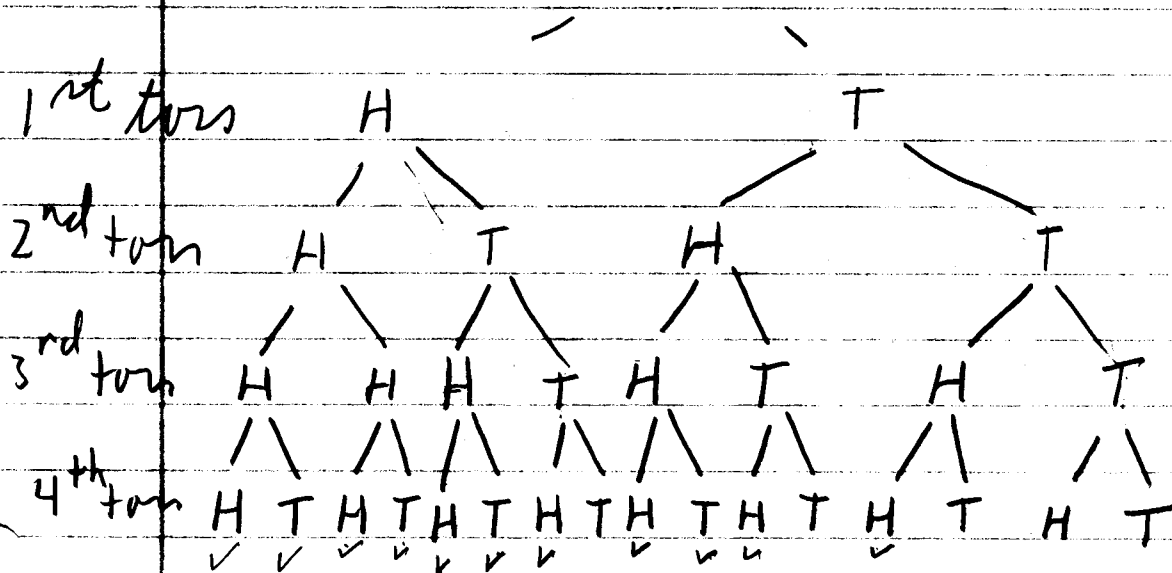


1) Draw a tree diagram



we want number of heads \geq number of tails
 Check the branches that satisfy this property: There are 11 out of 16. So

$$\text{prob}(\# \text{ of } H \geq \# \text{ of tails}) = \frac{11}{16} = .6875$$

2) Let X = number of students in the math club.

$$2X + \frac{X}{2} + \frac{X}{4} + 1 = 100$$

$$\frac{8X}{4} + \frac{2X}{4} + \frac{X}{4} = 99$$

$$\frac{11}{4}X = 99 \quad X = \frac{4}{11} \cdot 99 = 36$$

Let $y = \#$ of students in N. Mud. School

So $36 = 1 + \frac{y}{8}$ $y = 8 \cdot 35 = 280$

3. Date	Number of people who know
1/1	1
1/2	2
1/3	$4 = 2^2$
1/4	$2^3 = 8$
1/5	$2^4 = 16$
1/6	$2^5 = 32$
1/7	$2^6 = 64$
1/8	$2^7 = 128$ ← $\rightarrow 100$
1/9	$2^8 = 256$
1/10	$2^9 = 512$
1/11	$2^{10} = 1024$
1/12	$2^{11} = 2048$
1/13	$2^{12} = 4096$
1/14	$2^{13} = 8192$
1/15	$2^{14} = 16,384$
1/16	$2^{15} = 32,768$
1/17	$2^{16} = 65,536$
1/18	$2^{17} = 131,072$
1/19	$2^{18} = 262,144$
1/20	$2^{19} = 524,288$
	1/21 1,048,576

Some took the question to mean on
1/11, # of people who know

1	1	$= 2^1 - 1$
2	$1 + 2$	$= 2^2 - 1$
3	$1 + 2 + 2^2$	$= 2^3 - 1$
4	$1 + 2 + 2^2 + 2^3$	$= 2^4 - 1$
5	$1 + 2 + 2^2 + 2^3 + 2^4$	$= 2^5 - 1$
6	$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5$	$= 2^6 - 1 = 63$
7	$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$	$= 2^7 - 1 = 127$

So on 1/7/xx, in other words on day
early.

For $1,000,000 = 10^6$, 1/11/xx, the sum

of the people who know is $2^n - 1$. So

on 1/20/xx the sum of the people who
know is $2^{20} - 1 = 1,048,575$ & once
again this is just one day earlier.

Why?

$$(x-1)(1+x+\dots+x^n) = x+x^2+\dots+x^{n+1} - 1 - x - \dots - x^n$$

$$= x^{n+1} - 1. \text{ So}$$

$$1+x+\dots+x^n = \frac{x^{n+1}-1}{x-1}. \text{ Let } x=2, 1+2+\dots+2^n =$$

$$= 2^{n+1} - 1$$

2005

3. Using logs

If the date is January n , 2005 then

2^{n-1} is the number of people who know.

$$\text{So } 2^{n-1} = 100,$$

$$\ln(2^{n-1}) = \ln 100$$

$$(n-1) \ln 2 = \ln(100)$$

$$n-1 = \frac{\ln(100)}{\ln 2} = \frac{4.605170\dots}{.693147\dots} = 6.64$$

$n = 7.64$. We round this up to 8
So on Jan 8.

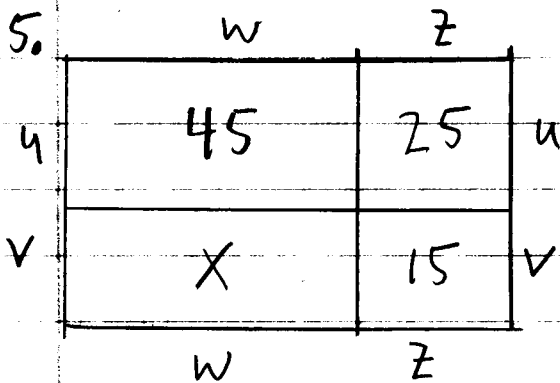
$$2^{n-1} = 10^6 \quad n-1 = \frac{6 \ln 10}{\ln 2} = 19.93$$

$n = 20.93$. So 1/21

4. The area of a circle is πr^2 , so the area of a semicircle is $\pi r^2/2$. So

$$\frac{\pi (12)^2}{2} - \pi \cdot 6^2 = \frac{\pi \cdot 6^2 \cdot 2^2}{2} - \pi \cdot 6^2 = \pi \cdot 6^2 \left(\frac{2^2}{2} - 1 \right)$$

$$= 36\pi(2-1) = 36\pi \approx 113.09 \dots \text{ (cm}^2\text{)}$$



$$uw = 45$$

$$uz = 25$$

$$wv = x$$

$$vz = 15$$

There are 2 ways to find the answer.

So $\frac{uw}{uz} = \frac{45}{25} = \frac{9}{5} \Rightarrow 5w = 9z$

$$\left. \begin{array}{l} 27z = zx \\ \end{array} \right\}$$

$$\frac{wx}{vz} = \frac{x}{15}$$

$$\Rightarrow \frac{15w = zx}{3(5w)}$$

$$x = 27$$

$$\frac{uz}{vz} = \frac{25}{15} = \frac{5}{3} \Rightarrow 3u = 5v$$

$$\left. \begin{array}{l} 9 \cdot 3u = u \cdot x \\ \end{array} \right\}$$

$$x = 27$$

$$\frac{uw}{wv} = \frac{45}{x}$$

$$\Rightarrow \frac{45v = u \cdot x}{9 \cdot 5v}$$