Level 2 : Part 2

1- 6 inches = $\frac{1}{6}$ of a yard

So the field is $110 \cdot 80 \cdot \frac{1}{6} = 1466.66$ cu. yds

Cost of the gravel $_{\text{+tax}}$ = $(9.37)(1466.66)$ + tax

$= \$\,13{,}742.66 + (.0625)(13742.66)$

$= \$14{,}601.58$ = Total gravel cost

How many truck loads are needed?

$$\frac{1466.66}{9} = 162.96 \simeq 163$$

Trucking cost $163 \cdot 25 = \$4075$

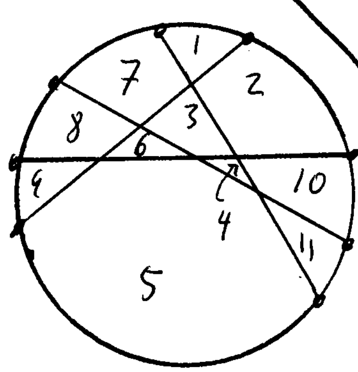
So 163 truck loads will be needed

& the total cost = 14,601.58 + 4075

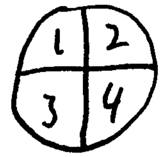
\$18,676.58 or \$18,676.51, if you use ~~1466.~~ 162.96 truck loads.

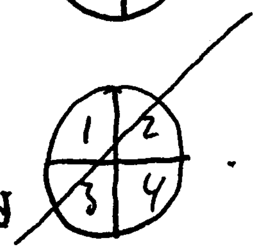
Both answers are ~~correct~~ acceptable.

2- 11 regions

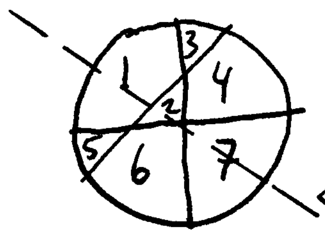


2 (cont) You did not have to prove 11 is the correct answer, but we will do it anyhow. One line cuts the circle into 2 pieces. 

. Another line that crosses the 1st line, cuts piece 1 into 2 pieces & piece 2 into 2 pieces. So now there are 4 pieces. 

. Now consider a line 

. It cuts piece 1 into 2, piece 2 into 2 pieces & piece 3 into 2 pieces. Thus 3 pieces have to be added. For a total of 7 pieces.



. Take a new line that meets all 3 of the old lines. Piece 1 is cut into 2 pieces, so is piece 2 & so is piece 6 (bad picture) & so is piece 7. So one have to add 4 pieces & 7+4=11.

n lines cut a circle into at most $(n^2+n+2)/2$ pieces

prime	$\frac{1}{p}$ decimal	period	smallest k $p \mid 10^k - 1$	period = k
3	$\frac{1}{3} = .\overline{3}$	1	$3 \mid 10^1 - 1$ $k=1$	Yes
7	$\frac{1}{7} = .\overline{142857}$	6	$7 \mid 10^6 - 1$ $k=6$	Yes
11	$\frac{1}{11} = .\overline{09}$	2	$11 \mid 10^2 - 1$ $k=2$	Yes
13	$\frac{1}{13} = .\overline{076923}$	6	$13 \mid 10^6 - 1$ $k=6$	Yes

Notice k does divide p-1

b) $k \mid 16$, so $k = 1, 2, 4, 8, 16$

c) $17 \nmid 10^1 - 1$ $17 \nmid (10^2 - 1)$ $17 \nmid (10^4 - 1)$ $17 \nmid (10^8 - 1)$

but $10^{16} - 1$ is so large the calculator does not display it as an integer. But we conclude $k = 16$.

d) My calculator give $\frac{1}{17} = .0588235294$, just 10 digits! We are going to have to divide by hand! or on another calculator I got

12 places $\frac{1}{17} \simeq .058823529412$

this is wrong!

$$17 \overline{\big)\, 1.0000000000000000}$$ quotient $.0588235294117647$

.85

150
136

140
136

40
34

60
51

90
85

50
34

160
153

70
68

20
17

30
17

130
119

110
102

80
68

120
119

1

So $\frac{1}{17} = .\overline{0588235294117647}$

4 - Let x = number of \$1 coins
y = # of quarters

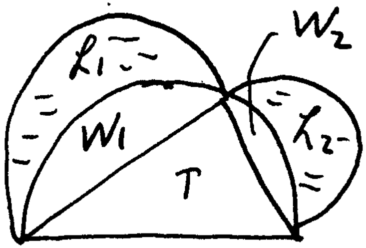
$x + y = 43$
$x + .25y = 29.50$. So $x = 43 - y$ &

so $43 - y + .25y = 29.50$
$-.75y = 29.50 - 43 = -13.5$
$y = \frac{13.5}{.75} = 18$

$x = 43 - 18 = 25$. So 25 \$1 coins &
~~quarters~~ 18 quarters

5 - Use the Pythagorean theorem. It states
if the length of AC is b; length of AB = c
& the length of CB = a, then $a^2 + b^2 = c^2$.
The area of $\mathcal{A} = \frac{1}{2}\pi b^2$
The area of $\mathcal{B} = \frac{1}{2}\pi a^2$
The area of $\mathcal{C} = \frac{1}{2}\pi c^2$. So
$\frac{1}{2}\pi a^2 + \frac{1}{2}\pi b^2 = \frac{1}{2}\pi(a^2 + b^2) = \frac{1}{2}\pi c^2$

5b)



Label the unshaded regions as above

Area of C = area(T) + area(W_1) + area(W_2)

= area of A + area of B = area of R_1 + area of W_1

+ area of R_2 + area of W_2. So

area(T) + area(W_1) + area(W_2) =

area(R_1) + area(W_1) + area(R_2) + area(W_2)

$\therefore$ area(T) = area(R_1) + area(R_2)