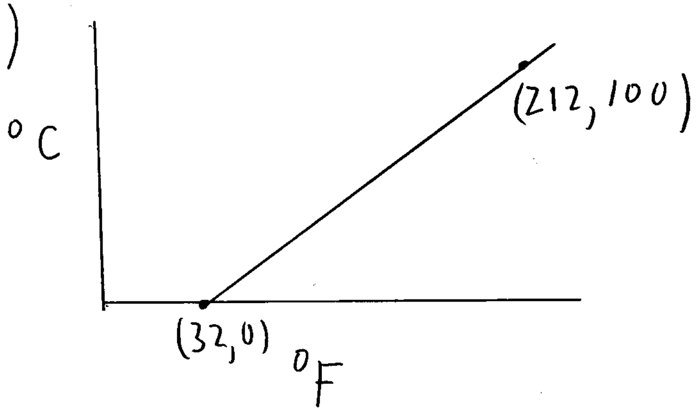
Level 2: Part 1

2004

1)

1	2	3
4	5	6
7	8	9

2a)



The slope $= \frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$

So $C - 0 = \frac{5}{9}(F-32)$ $\quad C = \frac{5F}{9} - \frac{160}{9}$

2b) $K = C + 273.16 = \frac{5F}{9} + 273.16 - \frac{160}{9} = \frac{5F}{9} + 255.4$

2c) So if is 0°F in Cincinnati, it is 255.4 K

If it is 3 times as cold on Mars the temperature is $(-3)(255.4) = -766.2$, but this is below absolute zero & hence is impossible.

Even if 3 times as cold means $255.4 - 3(255.4) = -2(255.4) = -510.8$ & this is below abs. 0.

3a) The distance to the moon in inches is

$238857 \cdot 5280 \cdot 12 = 15{,}133{,}979{,}520$ inches

The way to solve for x

$$2^x = 15133979520, \text{ so}$$

$$x = \frac{\ln(15133979520)}{\ln 2} = 33.81$$

Say 34 days or
Use the calculator by multiplying by 2
& do this 34 times.

3b) In one year, there are

$365.25 \cdot 24 \cdot 60 \cdot 60 = 31{,}557{,}000$ seconds in a yr.

So light will travel $(1.86)\cdot(10^5)\cdot(31{,}557{,}000)$
$= (5.8697136)10^{12}$ miles in a year. or
$(3.7190)10^{17}$ inches in a year. So 5.6 light
years is $(2.082)10^{18}$ inches.

$$2^x = (2.082)10^{18}$$

$$x = \frac{\ln(2.082) + 17}{\ln 2} = 60.85 \text{ days}$$ or

just 27 days after the plant got to the moon!

These problems can be done without logs, just divide (on the calculator) by 2 over & over.

4) Let y be the age of the youngest child & m the age of the man. Then the children have ages y, $y+g$, $y+2g$, $y+3g$, $y+4g$, $y+5g$, $y+6g$, $y+7g$, $y+8g$.

$$y(y+8g) = m+4$$

$$y^2 + (y+g)^2 + (y+2g)^2 + (y+3g)^2 + (y+4g)^2 + (y+5g)^2 + (y+6g)^2 + (y+7g)^2 + (y+8g)^2 = m^2$$

or

$$y(y+8g) = m+4$$

$$9y^2 + 2(g+2g+3g+4g+5g+6g+7g+8g)y + (1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2)g^2 = m^2$$

or

$$y(y+8g) = m+4$$

$$9y^2 + 2\cdot 36\cdot g\cdot y + 204g^2 = m^2$$

or

$$y(y+8g) = m+4$$

$$9\underbrace{(y^2+8gy)}_{=m+4} + 204g^2 = m^2$$

$$m^2 - 9m - 204g^2 - 36 = 0$$

$$m = \frac{9 \pm \sqrt{81 + 4(204g^2+36)}}{2}$$

$$= \frac{9 \pm \sqrt{816g^2 + 225}}{2}$$. We know

$g = 1, 2, 3$, since all the Bdays are today.

$g=1$ $m = 20.63$ $Y(Y+8) = 24.63 \Rightarrow Y = 2.37$

& the sum of squares condition does not work

$g=2$ $m = 34.03$ $Y^2 + 16Y - 38.03 = 0 \Rightarrow Y = 2.10$

- the sum of sq. does not work

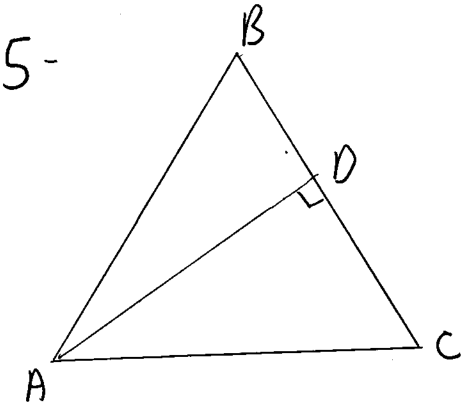
$g=3$ $m=48$ (this looks good, no decimal)

$0 = Y^2 + 24Y - 52 = (y+26)(Y-2) = 0$ $Y = 2$. So the

ages of the children are 2, 5, 8, 11, 14, 17, 20, 23, 26

$2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2 + 20^2 + 23^2 + 26^2 = 2304 = 48^2$.

5-



Since ΔABC is isosceles $\angle BAC = \angle ACB$. So (*) $2 \times \angle ACB + \angle ABC = 180$

$180 = 90 + \angle DAC + \angle ACB$.

$90 = \angle DAC + \angle ACB$. So

$180 = 2 \times \angle DAC + 2 \times \angle ACB$ (**)

So (*) = (**) means

$2 \times \cancel{\angle ACB} + \angle ABC = 2 \angle DAC + \cancel{2 \times \angle ACB}$

$\therefore \angle ABC = 2 \times \angle DAC$