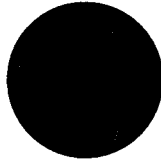


This exam has 5 questions, for a total of 100 points.

Write your answers in the Blue Book. Print your names & write the number of students taking this test in the upper right corner of the Blue Book. Put this test & the Blue Book in the provided envelope.

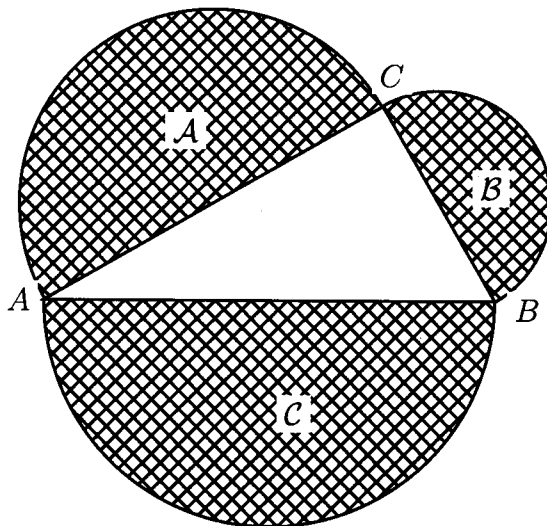
- (20 points) Your company is constructing a soccer field for a high school. The field is 110 yards long and 80 yards wide. To ensure adequate drainage, the Zoning Board requires 6 inches of gravel be laid before topsoil is added. The gravel comes in trucks that haul 9 cubic yards each & each cubic yard of gravel costs \$9.37. There is a 6.25% sales tax on the gravel and a \$25 per truck delivery charge and there is no sales tax on delivery charges. How many truck loads will be needed and how much will the entire project cost?
- (20 points) What is the largest number of regions that 4 lines can divide a disc into? The picture below shows that a disc can be divided into at most 4 regions with 2 lines. Line means straight line. You don't have to prove your answer is correct. Hint: The answer is  $> 9$ .



- The rational numbers can be regarded in 2 different ways. The first is that a rational number  $q$  is the quotient of integers  $a, b$  with  $b \neq 0$ . So  $q = a/b$ . Also a rational number can be viewed as an integer plus a decimal, which must eventually repeat. So  $2.7182818281828 \dots$  is rational with repeat 1828 and we say the repeat has length 4 or the period is 4. Note  $271801/99990 = 2.718281828 \dots$ 
  - (5 points) For each of the primes  $p = 3, 7, 11, 13$  find the length of the repeat of  $1/p$  and find the smallest integer  $k$  such that  $10^k - 1$  is a multiple of  $p$ . What are the connections between the period,  $k$  and  $p - 1$ ? Does  $k$  divide  $p - 1$ ?
  - (5 points) Based on your answer to the previous question, what are the possible lengths of repeat for  $p = 17$ ?
  - (5 points) Now compute  $10^k - 1$  for each of the possibilities you found in part (b). Does 17 divide  $10^k - 1$ ?
  - (5 points) Find the decimal expansion of  $1/17$ ?
- (20 points) In a parking meter there were 43 coins and each coin was a dollar coin or a quarter. The value of all of the coins is \$29.50. How many \$1 coins are there and how many quarters?

5. The triangle (pictured below) with vertices  $A$ ,  $B$  &  $C$  is a right triangle. The right angle occurs at vertex  $C$  or  $\angle ACB$  is a right angle. The regions are semidisks (half of a full disc) which is centered at the midpoint of each edge and of radius  $1/2$  of the length of the edge.

(a) (10 points) Prove the area of  $C$  is the area of  $A$  plus the area of  $B$ .



(b) (10 points) Prove

the area of  $\mathcal{L}_1$  + the area of  $\mathcal{L}_2$  = the area of  $T$ .

The crescent or new moon shapes  $\mathcal{L}_1$  &  $\mathcal{L}_2$  are called *lunes*, so you must prove the sum of the area of the lunes is the area of the right triangle  $T$ . Hint:  $AB$  is the diameter of the large circle,  $ACBD$ .

