

LEVEL I (advanced), part two March 2005

Instructions: Remember that you must explain your answers. Even correct answers without complete explanations and justifications may receive no credit! And even if you can't solve a problem completely, you should carefully explain what you have discovered about the problem since some partial credit may be awarded for your work.

1. It is easy to write an equation describing a curve that divides the plane into two pieces. For example, the equation $x^2 + y^2 = 1$ has a graph which is a circle: it has an inside and an outside — two pieces. Write an equation the graph of which divides the plane into *four* pieces.
2. Everybody knows that of all the rectangles with fixed area the square has the smallest perimeter. What about other polygons? Of all the triangles with fixed area, which has the smallest perimeter?
3. A certain number M is twice a perfect cube and 5 times a perfect square. What could M be?
4. One fourth the number of minutes from noon 'til now plus half the number of minutes from now 'til noon tomorrow is the time right (number of minutes past noon) now. What time is it?
5. Find a function whose derivative is $|x|^3$.