The 19th Annual UC Math Bowl

Level 3:Part 1 **Answer key** March 6, 2004

Answer the questions in the provided Blue Book. Write the number of students taking this test in the upper right corner of the Blue Book.

1. At *Annie's Home-Cooked Chicken Wings Restaurant*, chicken wings are served by the bucket. The **BIGGEST BUCKET O' WINGS** is really big! If the wings are removed two at a time, one wing will be left. If they are removed three at a time, two wings will remain. If they're removed four, five or six at a time, then three, four and five wings remain. If they are taken out seven at a time, no wings will remain. What is the smallest number of wings that could be in the **BIGGEST BUCKET O' WINGS**? How do you know?

   Answer- Let w be the number of wings in the Bucket. The conditions mean there are integers, k, l, j, n, m such that

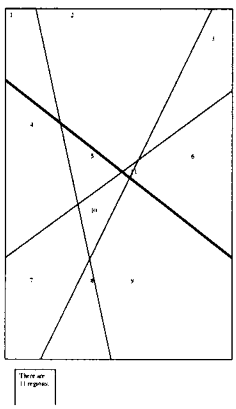
$$w = 2k + 1 \tag{1}$$
$$w = 3l + 2 \tag{2}$$
$$w = 4j + 3 \tag{3}$$
$$w = 5n + 4 \tag{4}$$
$$w = 6m + 5 \tag{5}$$

   Now start with equation (5) and just list the numbers of the form 6m+5. These are

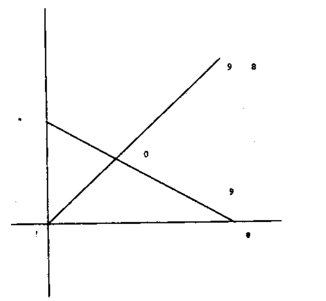
$$5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89, 95, 101, 107, 113, 119, 125, 131, 137, 143, 149 \cdots .$$

   Next find the numbers of the form $5n + 4$ that are on this list. These are 29,59,89,119,149$\cdots$. Next find the numbers of the form $4j + 3$ on this shortened list. This is 119, now it is odd, so it is of the form 2n+1. So 119=2(59)+1=3(39)+2=4(29)+3=5(23)+4=6(19)+5.

2. In the figure, the dark segment separates a rectangle into 2 regions. What is the greatest number of regions into which the rectangle can be separated by 4 segments? The answer is 11 regions.



3. A point is randomly selected from within a triangle with vertices at $(0, 0), (0, 4)$ & $(6, 0)$. What is the probability that the x-coordinate of the selected point is less than the y-coordinate?



Answer:

To find P we use $y = x$ and $y = -(2/3)x + 4$. So $x = -(2/3)x + 4$, $(5/3)x = 4$ and so $x = 12/5 = y$.

Now $y > x$ in $\triangle ABP$ which has area $\frac{1}{2} \cdot 4 \cdot \frac{12}{5} = \frac{24}{5} = 4.8$. The area of $\triangle ABC = 12$. So

$$\text{Prob}(y > x) = \frac{\frac{24}{5}}{12} = \frac{\frac{24}{5}}{\frac{12}{1}} = \frac{24}{5}\frac{1}{12} = \frac{2}{5} = .4.$$

In percent, 40%.

4. Three big ducks went out one day
With two ducklings behind.
Fourteen kilograms was their weight
When all five were combined.

Add one duck and one duckling
And their weight increased.
Nineteen kilograms was the total
Of the seven with webbed feet.

If all ducklings have one weight,
and ducks have another,
Then you should find the total weight
of three-one child, two mothers

Anonymous

Answer- Let D be the weight of a big duck and d be the weight of a little duck. Then 3D+2d=14 and D+d=5, so 2D+d=9

5. Bugs B. has carrots of length 10 cm, 12 cm and 15 cm. How can he use these carrots to mark a length of 17 cm? 12+15-10=17