University of Cincinnati Math Competition – 2004
Level II, Answers for Part 1

1. 8’8” = 104”

   104” – 24” = 80” length of one side of tiled region

   Let x = length of other side of tiled region

   $A_{tiled} = 80x \text{ in}^2$

   $A_{kitchen} = 104(x + 24)$

$$\frac{80x}{104(x+24)} = \frac{2}{3}$$
$$240x = 208(x + 24)$$
$$240x = 208x + 208(24)$$
$$32x = 208(24)$$
$$x = \frac{208(24)}{32} = 156$$

   Kitchen is 180” by 104”.

   Area to be tiled is 156” by 80” = 12480 in^2

   Area of each tile is 48 in^2

   Number of tiles needed = $\frac{12480}{48} = 260$ or 10 rows of 26 tiles each to fit perfectly

   Total cost = 260(.59)(1.0675) = 163.7545 $\Rightarrow$ \$163.75

2. The terms of the sequence in general form will be:

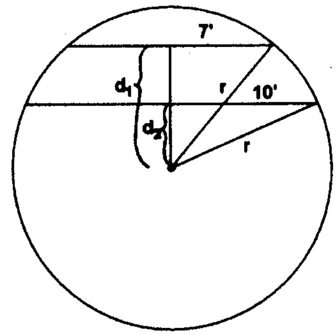
   3, b, 2b + 3, 5b + 6, 12b + 15, 29b + 36, 70b + 87, 169b + 210, 408b + 507

$$985b + 1224 = 8119$$
$$b = 7$$

3.

Let the planes cut the sphere at distances d_1 and d_2 from the center where $d_1 > d_2$. If R is the radius of the sphere,

$$R^2 - d_1^2 = 7^2;$$
$$R^2 - d_2^2 = 10^2;$$
$$d_1^2 - d_2^2 = 51.$$



$$(d_1 - d_2)(d_1 + d_2) = 51$$

Since $d_1 - d_2 = 2$, $d_1 + d_2 = \frac{51}{2}$.

Solving these equations $d_1 = \frac{55}{4}$; $d_2 = \frac{47}{4}$.

Using d_1,

$$R = 7^2 + \left(\frac{55}{4}\right)^2 = 15.43 \text{ ft.}$$

4. If x is the radius of the given pile at its base:

$$3016 = \frac{20\pi x^2}{3}$$
$$x = \sqrt{\frac{9048}{20\pi}} \approx 12.00012$$
$$\frac{x'}{h'} = \frac{12.00012}{20} = 0.6$$
$$x' = 0.6h'$$
$$4016 = \left(\frac{\pi}{3}\right)(x')^2 h' = \left(\frac{\pi}{3}\right)(0.36h'^2)h' \approx 0.37699h'^3$$
$$h' = \sqrt[3]{10652.80246} \approx 22.00330$$

The pile height changes by 2.0033 ft.

5. (a).

$$x\$(2\$3) = 3$$
$$x\$(2^3 - 6) = 3$$
$$x\$2 = 3$$
$$x^2 - 2x = 3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$

(b). This is equivalent to asking for what value of k does the equation not have a solution. Using the discriminant, no solutions would occur when

One could also use the graphing calculator and graph the parabola, , locate the vertex at (1, −1) and conclude that if the value of y is less than −1 then y = k does not intersect the parabola.

(c). If a$2=2$a then a^2=2^a, so a=2,4 or about -.7666646956212....